Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third Semester B.Tech Degree (S,FE) Examination January 2022 (2015 Scheme)

Course Code: MA201

Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer any two full questions, each carries 15 marks Marks

1 a) Define continuity of a complex valued function f(z) at a point z = z_0. Also check whether the function f(z) = $\begin{cases} \frac{\text{Im}(z^2)}{|z|^2} & , z \neq 0 \\ 0 & z = 0 \end{cases}$ is continuous at z = 0. (7)

b) Check whether U(x,y) = x^3 - $3xy^2$ + $3x^2$ - $3y^2$ + 1 is harmonic .If so find its harmonic conjugate. (8)

2 a) Show that if f(z) = u+iv is an analytic function with constant modulus, then f is a constant. (8)

b) Write the real and imaginary parts of the transformation f(z) = $\frac{1+iz}{1-iz}$. Also find the image of |z|<1 under this transformation. (7)

3 a) i) Find the image of the circle $x^2 + y^2 - 6 = 0$ under the transformation w = $\frac{1}{z}$. (7)

ii) Find the image of the strip 1< y < 2 under the transformation w = sinz

b) Find the bilinear transformation which maps (-1, -i, 0) to (0, -i, 2). Also find the critical points of this transformation. (8)

PART B

Answer any two full questions, each carries 15 marks

4 a) Evaluate $\int_0^{3+i} \bar{z}^2 dz$ along (8)

a) the line x = 3y

b) along z(t)=3t+it^2

b) Using Cauchy's integral formula Evaluate the integral $\int_C \frac{e^z}{(2z-1)^2}\, dz$ over the circle |z|=1. (7)

5 a) Find the singular points and residue at singular point of the function f(z) = $\frac{tanz}{z^2+1}$ which lie inside the circle |z| = 3/2. (7)

b) Find the Laurent series of $f(z) = \frac{1}{z(z-2)(z-5)}$ valid in the region 2 < |z| < 5 around z = 0. (8)

6 a) Using residue theorem, evaluate $\int_C \frac{dz}{(z^2+4)^2}$ over the circle |z-i|=2. (7)

b) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+3isin\theta}$. (8)

PART C

Answer any two full questions, each carries 20 marks

7 a) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$. (8)

b) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 2 \\ -1 & -2 & 3 \\ 2 & 4 & 0 \end{bmatrix}$ (6)

c) Test for consistency and solve the following system (6)

x – y + z = 1

2x + y – z = 2

5x - 2y + 2z = 5.

8 a) Find the basis and dimension of raw space and column space of the matrix $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ (8)

b) Let A=$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix}$ find AA^T and A^TA and their traces. (6)

c) Check whether the vectors (1,1,1), (1, -1,1), (1,1, -1) are linearly independent. (6)

9 a) Write down the matrix associated with the quadratic form $3x^2 + 3y^2 + 2xy = 1$. Also convert it to canonical form and find the corresponding transformation. (8)

b) Diagonalize the matrix M=$\begin{bmatrix} 3 & 0 & 0 \\ -3 & 4 & 9 \\ 0 & 0 & 3 \end{bmatrix}$. (12)

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